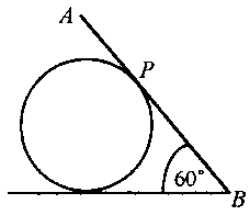
MECHANICS (C) UNIT 2 **TEST PAPER 10**

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. Particles of mass $2m$, $3m$ and $5m$ are placed at the points in the x-y plane with coordinates $(-1, 5)$, $(0, 6)$ and $(3, -2)$ respectively.
   Find the coordinates of the centre of mass of this system of particles. [4]

2. A lorry of mass 3800 kg is pulling a trailer of mass 1200 kg along a straight horizontal road. At a particular moment, the lorry and trailer are moving at a speed of 10 ms^{-1} and accelerating at $0{\cdot}8$ ms^{-2}. The resistances to the motion of the lorry and the trailer are constant and of magnitude 1600 N and 600 N respectively.
   Find the rate, in kW, at which the engine of the lorry is working. [4]

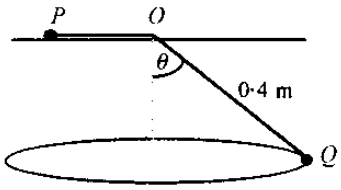
3. A bullet of mass $0{\cdot}05$ kg is fired with speed u ms^{-1} from a gun, which recoils at a speed of $0{\cdot}008u$ ms^{-1} in the opposite direction to that in which the bullet is fired.
   (i) Find the mass of the gun. [2]
   (ii) Find, in terms of u, the kinetic energy given to the bullet and to the gun at the instant of firing. [3]
   (iii) If the total kinetic energy created in firing the gun is 5103 J, find the value of u. [2]

4. 

   A uniform plank AB, of mass 3 kg and length 2 m, rests in equilibrium with the point P in contact with a smooth cylinder. The end B rests on a rough horizontal surface and the coefficient of friction between the plank and the surface is $\frac{1}{3}$. AB makes an angle of 60° with the horizontal.
   If the plank is in limiting equilibrium in this position, find
   (i) the magnitude of the force exerted by the cylinder on the plank at P, [6]
   (ii) the distance AP. [3]

5. Two smooth spheres A and B have equal radii and masses $0{\cdot}4$ kg and $0{\cdot}8$ kg respectively. They are moving in opposite directions along the same straight line, with speeds 3 ms^{-1} and 2 ms^{-1} respectively, and collide directly. The coefficient of restitution between A and B is $0{\cdot}8$.
   (i) Calculate the speeds of A and B after the impact, stating in each case whether the direction of motion has been reversed. [8]
   (ii) Find the kinetic energy, in J, lost in the impact. [2]

6. A particle P, of mass 0·5 kg, rests on the surface of a rough horizontal table. The coefficient of friction between P and the table is 0·5. P is connected to a particle Q, of mass 0·2 kg, by a light inextensible string passing through a small smooth hole at a point O on the table, such that the distance OQ is 0·4 m. Q moves in a horizontal circle while P remains in limiting equilibrium.



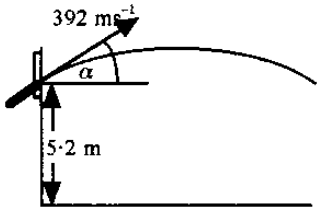
(i) Calculate the angle θ which OQ makes with the vertical. [4]

(ii) Show that the speed of Q is 1·33 ms^{-1}. [3]

The motion is now altered so that Q hangs at rest below O and P moves in a horizontal circle on the table with speed 0·84 ms^{-1}, at a constant distance r from O but tending to slip away from O.

(iii) Find the value of r. [5]

7. A bullet is fired out of a window at a height of 5·2 m above horizontal ground. The initial velocity of the bullet is 392 ms^{-1} at an angle α above the vertical, where $\sin\alpha = \frac{1}{20}$, as shown.



Find

(i) the range of times after firing during which the bullet is 15 m or more above ground level, [5]

(ii) the greatest height above the ground reached by the bullet, [3]

(iii) the horizontal distance travelled by the bullet before it reaches its highest point. [2]

Certain modelling assumptions have been made about the bullet.

(iv) State these assumptions and suggest a way in which the model could be refined. [2]

(v) State, with a reason, whether you think this refinement would make a significant difference to the answers. [2]

MECHANICS 2 (C) TEST PAPER 10 : ANSWERS AND MARK SCHEME

1. $2m(-1, 5) + 3m(0, 6) + 5m(3, -2) = 10m(\bar{x}, \bar{y})$ $(1{\cdot}3, 1{\cdot}8)$ M1 M1 A1 A1 4

2. $P = (2200 + 5000 \times 0.8)v = 6200 \times 10 = 62$ kW M1 A1 M1 A1 4

3. (i) Momentum : $0{\cdot}05u = M(0{\cdot}008u)$ $m = 6{\cdot}25$ kg M1 A1
   (ii) K.E. given to bullet $= \frac{1}{2}(0{\cdot}05)u^2 = \frac{1}{40}u^2$ J B1
   K.E. given to gun $= \frac{1}{2}(6{\cdot}25)(0{\cdot}008u)^2 = \frac{1}{5000}u^2$ J M1 A1
   (iii) $u^2(\frac{1}{40} + \frac{1}{5000}) = 5103$ $u = 450$ M1 A1 7

4. (i) Resolve vert : $3g = R + \frac{1}{2}S$ Resolve horiz : $\frac{1}{3}R = \frac{\sqrt{3}}{2}S$ M1 A1 A1
   Hence $3g = \frac{1}{2}(3\sqrt{3} + 1)S$ $S = 6g/(3\sqrt{3} + 1) = 9{\cdot}49$ N A1 M1 A1
   (ii) M(B) : $3g/2 = Sd$ $d = 1{\cdot}55$ m $AP = 0{\cdot}45$ m M1 A1 A1 9

5. (i) Momentum : $1{\cdot}2 - 1{\cdot}6 = 0{\cdot}4v_A + 0{\cdot}8v_B$ $v_A + 2v_B = -1$ M1 A1
   Elasticity : $(v_B - v_A)/(-2 - 3) = -0{\cdot}8$ $v_A - v_B = -4$ M1 A1
   Solve : $v_A = -3$, $v_B = 1$ M1 A1 A1
   A has speed 3 ms^{-1}, B has speed 1 ms^{-1}, both directions reversed A1
   (ii) K.E. lost $= 0{\cdot}2(9) + 0{\cdot}4(4) - 0{\cdot}2(9) - 0{\cdot}4(1) = 1{\cdot}2$ J M1 A1 10

6. (i) $T = F = \mu R$, so $T = \frac{1}{4}g$ $T\cos\theta = 0{\cdot}2g$ B1 M1 A1
   $\cos\theta = 0{\cdot}8$ $\theta = 36{\cdot}9°$ A1
   (ii) $T\sin\theta = 0{\cdot}2v^2/(0{\cdot}4\sin\theta)$ $v^2 = 0{\cdot}5g\sin^2\theta = 1{\cdot}764$ M1 A1
   $v = \sqrt{1{\cdot}764} = 1{\cdot}33$ ms^{-1} A1
   (iii) Now $T = 0{\cdot}2g$ $0{\cdot}2g + 0{\cdot}25g = 0{\cdot}5v^2/r$ B1 M1 A1
   $0{\cdot}45g = 0{\cdot}5(0{\cdot}84^2)/r$ $r = 0{\cdot}08$ M1 A1 12

7. (i) $y = (392\sin\alpha)t - 4{\cdot}9t^2 = 19{\cdot}6t - 4{\cdot}9t^2$ B1
   15 m above ground, $y = 9{\cdot}8$ $19{\cdot}6t - 4{\cdot}9t^2 = 9{\cdot}8$ M1
   $t^2 - 4t + 2 = 0$ $(t-2)^2 - 2 = 0$ $t = 2 \pm \sqrt{2}$ M1 A1
   Times are from 0·586 s to 3·41 s A1
   (ii) y is maximum when $19{\cdot}6 - 9{\cdot}8t = 0$ $t = 2$ $y = 19{\cdot}6$ M1 A1
   Height above ground = 24·8 m A1
   (iii) $x = (392\cos\alpha)t = 391{\cdot}5 \times 2 = 783$ m M1 A1
   (iv) Bullet = particle; assumed no air resistance Include this B1 B1
   (v) As bullet is small and moving fast, probably little difference B1 B1 14